Radar Imaging of Breast Cancer Using Kirchhoff Migration and Singular Value Decomposition

Takuya Sakamoto
Grad. School of Engineering, University of Hyogo
Himeji, Hyogo 671-2280, Japan
Grad. School of Informatics, Kyoto University
Kyoto, Kyoto 606-8501, Japan
Dept. of Electrical Engineering, University of Hawaii at Manoa
Honolulu, HI 96822, USA

Hang Song and Takamaro Kikkawa
Research Center for Nanodevioces and Systems
Graduate School of Advanced Sciences of Matter
Hiroshima University
Higashihiroshima, Hiroshima 739-8527, Japan

***Abstract*—This paper presents a radar imaging technique for breast cancer detection using an ultra-wideband radar. The proposed technique employed modified Kirchhoff migration to achieve high-resolution imaging. In addition, a singular value decomposition was used for noise suppression to obtain a clear image. The proposed technique was applied to experimental data measured using an ultra-wideband array radar system and breast cancer phantom.**

***Index Terms*—ultra-wideband radar, imaging, breast cancer, Kirchhoff migration**

I. Introduction

Breast cancer imaging using a radar system is a promising modality for detecting and monitoring cancer tumors because of its simplicity and low cost compared with other systems such as X-ray mammography and magnetic resonance imaging [1]-[4]. Unlike X-ray mammography systems, ultra-wideband radar imaging can produce three-dimensional tomographic images, and does not cause discomfort or pain due to breast compression [5]. In addition, radar systems do not use ionizing radiation [6]. Recently, a high-resolution radar imaging technique using modified Kirchhoff migration has been proposed and its high-resolution imaging capability has been demonstrated in the field of airport body scanner systems [7], [8]. In the present paper, we applied modified Kirchhoff migration and a noise suppression technique using a singular value decomposition (SVD) with experimental data measured using an ultra-wideband radar system consisting of integrated circuits developed by Hiroshima University [9]-[13].

II. System Model

We use an ultra-wideband radar with center frequency of 3.3 GHz and -10-dB bandwidth of 2.9 GHz. The system has eight transmitting and eight receiving antennas, forming an 4×4 multiple-input multiple-output (MIMO) antenna array (see Fig. 1). The entire array is mechanically rotated from 0 to 360 deg with intervals of 3 deg, and effectively functions as a virtual large-scale antenna array. Figure 2 shows a measurement setup with two cancer tumor phantoms embedded in a breast phantom, whose permittivity is $\varepsilon_{\mathrm{r}} = 6$. The cancer tumor targets have a cubic shape with size $10 \times 10 \times 10\ \mathrm{mm}^3$.

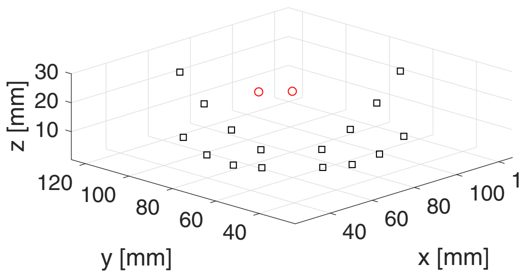

Fig. 1. Array antenna configuration.

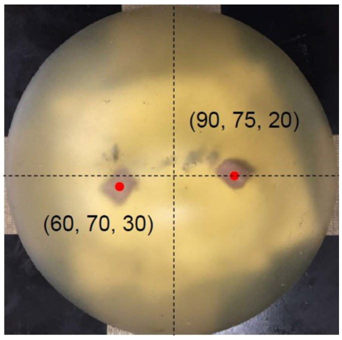

Fig. 2. Breast and cancer tissue phantom used for measurement.

The targets are located at (60 mm, 70 mm, 30 mm) and (90 mm, 75 mm, 20 mm), respectively.

III. Proposed Imaging Algorithm

A. Modified Kirchhoff Migration

The general Kirchhoff integral is derived from Green theorem for scalar fields $f(\boldsymbol{r})$ and $g(\boldsymbol{r})$ defined in a three-dimensional position $\boldsymbol{r}$

$$\oint_{\partial V}\{f\nabla g - g\nabla f\}\cdot \mathrm{d}\boldsymbol{n} = \int_V \{f\nabla^2 g - g\nabla^2 f\}\mathrm{d}V. \quad (1)$$

If g is a Green function of Poisson's equation $\nabla^2 f(\boldsymbol{r}) = u(\boldsymbol{r})$ as $\nabla^2 g = \delta(\boldsymbol{x})$, Eq. (1) is rewritten as

$$f(\boldsymbol{x}) = \oint_{\partial V}\{f\nabla g - g\nabla f\}\cdot \mathrm{d}\boldsymbol{n} + \int_V \{g\nabla^2 f\}\mathrm{d}V. \quad (2)$$

The modified Kirchhoff migration is derived from this expression and the reciprocity theorem, and the image $I(\boldsymbol{r})$ is calculated as

$$\begin{aligned} I(\boldsymbol{r}) &= \int_{S_1}\int_{S_2} \frac{\partial R_1}{\partial n_1}\frac{\partial R_2}{\partial n_2}\frac{1}{R_1 R_2} \\ &\cdot \left\{\frac{1}{c^2}\frac{\partial^2}{\partial t^2}s_0(\boldsymbol{r}_1,\boldsymbol{r}_2,t+\tau)\right. \\ &+ \frac{1}{c}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{\partial}{\partial t}s_0(\boldsymbol{r}_1,\boldsymbol{r}_2,t+\tau) \\ &+ \left.\frac{1}{R_1 R_2}s_0(\boldsymbol{r}_1,\boldsymbol{r}_2,t+\tau)\right\}\mathrm{d}S_1\mathrm{d}S_2\Big|_{t=0}, \quad (3) \end{aligned}$$

where S_1 and S_2 are the transmitting and receiving scan surfaces, respectively, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the positions of transmitting and receiving elements, $R_1 = |\boldsymbol{r}_1 - \boldsymbol{r}|$ and $R_2 = |\boldsymbol{r}_2 - \boldsymbol{r}|$ are propagation path length, n_1 and n_2 are parameters along the normal vectors of S_1 and S_2, c is the speed of the radio wave in the medium, $s_0(\boldsymbol{r}_1,\boldsymbol{r}_2,t)$ is the signal for the transmitting element at $\boldsymbol{r}_1$ and receiving element at $\boldsymbol{r}_1$, and $\tau = (R_1 + R_2)/c$ is a delay time [7], [8].

B. Noise Suppression Using SVD

Modified Kirchhoff migration is known to emphasize high frequency components to achieve better spatial resolution. This process is effective when the signal-to-noise ratio (S/N) is sufficiently high. In contrast, for signals with low S/N, the technique emphasizes noise, compromising image quality. To resolve this issue of modified Kirchhoff migration, we employed a SVD-based noise suppression technique.

We applied the SVD to radar data for each antenna pair to suppress noise and obtain clearer images. The antenna array was rotated every three degrees during the measurement, generating signals $s_{i,j}(t,\theta)$ in the time domain for the rotation angle θ and the i-th and j-th antennas. The frequency-domain representation is obtained as

$$S_{i,j}(\omega,\Omega) = \int\int s_{i,j}(t,\theta)\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{e}^{-\mathrm{j}\Omega\theta}\mathrm{d}t\mathrm{d}\theta. \quad (4)$$

The signal in the frequency domain $S_{i,j}(\omega,\Omega)$ is expressed as a $m\times n$ matrix S with its rows and columns corresponding to Ω and ω. We assume $m \geq n$. In this study, $m = 1024$ and $n = 120$, specifically. We apply the SVD to S, where subscripts i, j are omitted for simplicity, and obtain a factorization form

$$\begin{aligned} S &= U\Sigma V^{\mathrm{H}}, \\ &= \textstyle\sum_{i=1}^{n}\sigma_i \boldsymbol{u}_i\boldsymbol{v}_i^{\mathrm{H}} \end{aligned} \quad (5)$$

where U is an $m\times m$ complex unitary matrix, Σ is an $m \times n$ non-negative real-valued diagonal matrix called a singular value matrix, and V is an $n\times n$ complex unitary matrix. The singular value matrix Σ has entries of real-valued singular values $\sigma_1,\sigma_2,\cdots,\sigma_n$, and $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are the i-th left- and right-singular vectors.

We suppress noise contained in S and obtain $\bar{S}$ for a constant $k > 1$ as

$$\bar{S} = U\Sigma^k V, \quad (6)$$

which corresponds to a process for emphasizing large singular values, while suppressing small singular values. Specifically $k = 2$ was selected in this study, but it will be important for future studies to investigate optimization of the k value depending on the S/N. Because small singular values correspond to noise components, this process can suppress noise contained in the received signal. This SVD-based approach does not affect the spatial resolution of images because this method does not involve a smoothing process unlike many conventional techniques. Finally, inverse Fourier transform is applied to the signal $\bar{S}$ as

$$\bar{s}_{i,j}(t,\theta) = \int\int \bar{S}_{i,j}(\omega,\Omega)\mathrm{e}^{\mathrm{j}\omega t}\mathrm{e}^{\mathrm{j}\Omega\theta}\mathrm{d}\omega\mathrm{d}\Omega, \quad (7)$$

and the noise-suppressed $\bar{s}_{i,j}(t,\theta)$ is used for modified Kirchhoff migration instead of $s_{i,j}(t,\theta)$.

IV. Performance Evaluation of the Proposed Imaging Technique

First, we applied conventional delay-and-sum (DAS) migration and modified Kirchhoff migration to the measured data without SVD-based noise suppression, and evaluated the imaging capability of each method. Figures 3 and 4 show the images generated using DAS and modified Kirchhoff migration, respectively. These images were formed by rendering the -3-dB contour surface of the 3-dimensional image normalized to the maximum value. The results revealed that modified Kirchhoff migration achieved higher resolution, making the target image size smaller. However, modified Kirchhoff migration uses the first and second derivatives of the received signals, which emphasizes high-frequency components including noise, resulting in the noisy target image shown in Fig. 4.

Figure 5, 6 and 7 show the power spectrum density functions of the raw signal $s(t,\theta)$, the first derivative $\partial/\partial t s(t,\theta)$ and the second derivative $\partial^2/\partial t^2 s(t,\theta)$, where all power spectrum density functions of antenna pairs and rotation angles are averaged. These figures confirmed that the first and second derivatives emphasize higher frequency components, while the frequency bandwidths are smaller than the raw signal. This means that the range resolution is sacrificed to enhance the cross-range resolution. This presents a trade-off between the range and cross-range resolutions, for which an appropriate balance is made by modified Kirchhoff migration. Nonetheless, the formula underlying modified Kirchhoff migration does not consider the effect of noise in imaging.

Next, we applied modified Kirchhoff migration with an SVD-based noise suppression technique. Figure 8 shows an image generated using the proposed technique. The contours of the targets were both smooth, while the size of the target

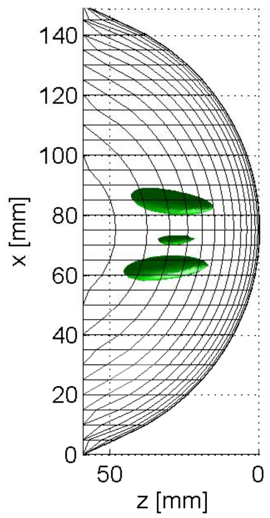

Fig. 3. Image generated using DAS migration.

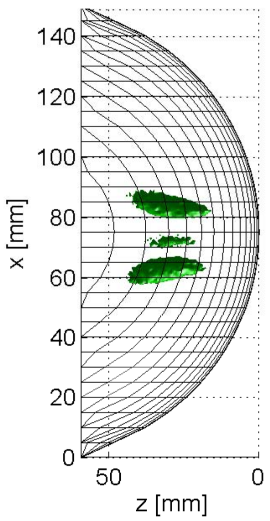

Fig. 4. Image generated using modified Kirchhoff migration.

images was not sacrificed because the proposed technique does not use smoothing to suppress noise. The results revealed that the image size of the tagets in the z-direction was improved by 19% on average using modified Kirchhoff migration compared with conventional DAS migration. This suggests the effectiveness of the proposed technique in realizing high-resolution imaging, even from noisy data.

V. Conclusion

The current paper presented a breast cancer imaging technique using modified Kirchhoff migration and an SVD-based noise suppression technique. We measured two cancer tumor phantoms in a breast phantom using an ultra-wideband radar system with an 4×4 MIMO antenna array. The entire

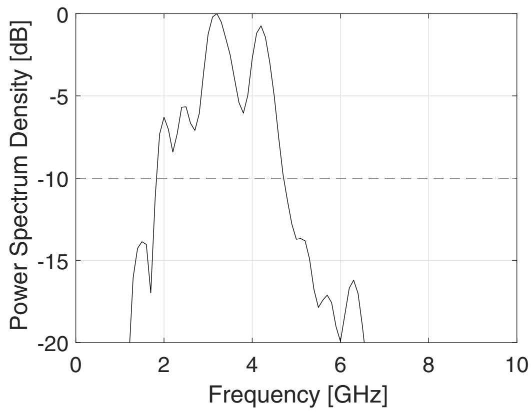

Fig. 5. Averaged power spectrum density function of the received signals.

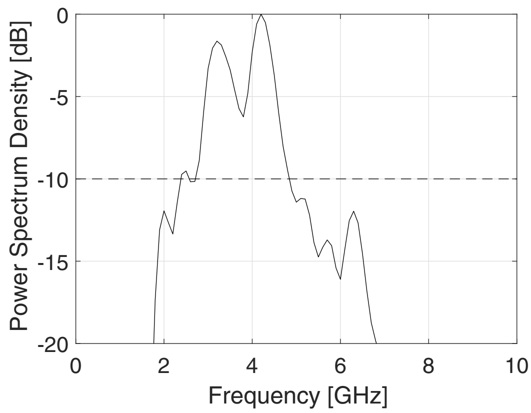

Fig. 6. Averaged power spectrum density function of the first derivative of the received signals.

array was rotated 360 degrees with intervals of 3 degrees. The frequency-domain signals were decomposed to obtain singular values, and small singular values corresponding to noise were suppressed. The de-noised data were used for the modified Kirchhoff migration, to calculate a three-dimensional tomographic image. The results indicated that the target image was better-focused in the cross-range direction (z-axis) using the proposed technique, because higher-frequency components are emphasized by modified Kirchhoff migration. In addition, the SVD-based noise suppression was demonstrated to be effective in generating clearer images while the image resolution remained unchanged.

Acknowledgment

This study was supported in part by the Research Center for Biomedical Engineering, KAKENHI grants from the Japan Society for the Promotion of Science (25249057, 15K18077 and 15KK0243) and the Center of Innovation Program of Kyoto University. We thank Benjamin Knight, MSc, from Edanz Group (www.edanzediting.com/ac) for editing a draft of this manuscript.

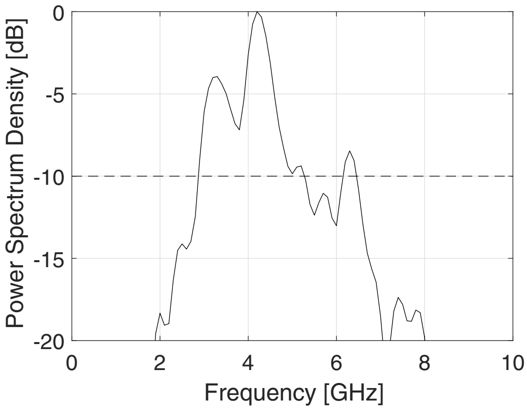

Fig. 7. Averaged power spectrum density function of the second derivative of the received signals.

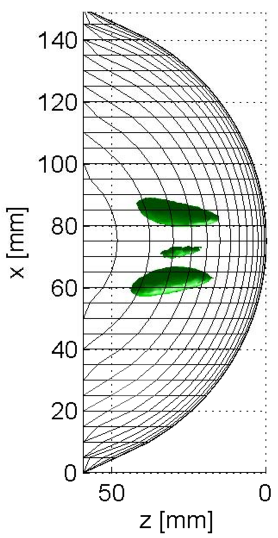

Fig. 8. Image generated using modified Kirchhoff migration and noise suppression with singular value decomposition.

REFERENCES

[1] M. O'Halloran, E. Jones, and M. Glavin, "Quasi-multistatic MIST beamforming for the early detection of breast cancer," *IEEE Trans. Biomed. Eng.,* vol. 57, no. 4, pp. 830–840, doi: 10.1109/TBME.2009.2016392, 2010.

[2] L. M. Neira, R. O. Mays, and S. C. Hagness, "Human breast phantoms: Test beds for the development of microwave diagnostic and therapeutic technologies," *IEEE Pulse,* vol. 8, no. 4, pp. 66–70, doi: 10.1109/MPUL.2017.2701489, 2017.

[3] R. O. Mays, N. Behdad, and S. C. Hagness, "Array sensitivity for model-based microwave breast imaging," *IEEE Trans. Antennas and Propag.,* vol. 65, no. 6, pp. 2958–2965, doi: 10.1109/TAP.2017.2688800, 2017.

[4] T. J. Colgan, S. C. Hagness, and B. D. Van Veen, "A 3-D level set method for microwave breast imaging," *IEEE Trans. Biomed. Eng.,* vol. 62, no. 10, pp. 2526–2534, doi: 10.1109/TBME.2015.2435735, 2015.

[5] R. O. Mays, N. Behdad, and S. C. Hagness, "Array sensitivity for model-based microwave breast imaging," *IEEE Trans. Antennas Propag.,* vol. 65, no. 6, pp. 2958–2965, 2017.

[6] H. Song, H. Kono, Y. Seo, A. Azhari, J. Somei, E. Suematsu, Y. Watarai, T. Ota, H. Watanabe, Y. Hiramatsu, A. Toya, X. Xiao, and T. Kikkawa, "A radar-based breast cancer detection system using CMOS integrated circuits,"*IEEE Access,* vol. 3, pp. 2111–2121, 2015.

[7] T. Sakamoto, T. Sato, P. J. Aubry, and A. G. Yarovoy, "Ultra-wideband radar imaging using a hybrid of Kirchhoff migration and Stolt F-K migration with an inverse boundary scattering transform," *IEEE Trans. Antennas Propag.,* vol. 63, no. 8, pp. 3502–3512, 2015.

[8] X. Zhuge, A. G. Yarovoy, T. Savelyev, and L. Ligthart, "Modified Kirchhoff migration for UWB MIMO array-based radar imaging," *IEEE Trans. Geosci. Remote Sens.,* vol. 48, no. 6, pp. 2692–2703, 2010.

[9] H. Song, H. Sato, X. Xiao, and T. Kikkawa, "A portable breast cancer imaging system with cross-shaped dome antenna array," *Proc. 11th European Conference on Antennas and Propagation,* pp. 3474–3475, doi: 10.23919/EuCAP.2017.7928284, 2017.

[10] A. Azhari, K. Yuki, X. Xiao, and T. Kikkawa, "A Tx/Rx 3-20-GHz DP16T switching matrix for breast cancer detection system," *Proc. 2016 IEEE MTT-S Latin America Microwave Conference,* doi: 10.1109/LAMC.2016.7851283, 2016.

[11] H. Song, H. Kono, Y. Seo, A. Azhari, X. Xiao, and T. Kikkawa, "Confocal imaging by turning antennas with CMOS integrated circuits for breast cancer detection," *Proc. 2016 International Symposium on Antennas and Propagation,* pp. 920–921, 2016.

[12] T. Hirano and T. Kikkawa, "4×4 Pentagonal patch array antenna for breast cancer detection," *Proc. 2016 IEEE Conference on Antenna Measurements & Applications,* doi: 10.1109/CAMA.2016.7815783, 2016.

[13] H. Song, S. Kubota, X. Xiao, and T. Kikkawa, "Design of UWB antennas for breast cancer detection," *Proc. 2016 International Conference on Electromagnetics in Advanced Applications,* pp. 321–322, doi: 10.1109/ICEAA.2016.7731386, 2016.