# Experimental Study on Imaging Algorithm with Simple UWB Radar for a Target with Translation and Rotation 

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#### Abstract

Developing a reliable surveillance system is important to resolve the social concern of terrorist activities. Radar systems can offer additional functions to conventional camerabased surveillance systems. Most conventional radar imaging algorithms employ a large-scale antenna array to obtain a high-resolution image, which makes the practical use of radar imaging technology prohibitively expensive. A simplified radar system with only three antennas has been proposed to estimate the translatory motion of a target. However, this method assumes that the target only undergoes translation and no rotation, which is unrealistic. This paper proposes a new method to estimate the translation and rotation of a target even though the target shape is unknown. The proposed method employs five antennas to optimize an assumed elliptical model using the data recorded by the radar system. The proposed method is applied to experimental data to demonstrate its effectiveness in a realistic environment.


## 1. INTRODUCTION

There is a great demand for the development of a cost-effective and reliable surveillance system to prevent criminal and terrorist activities. Conventional systems are mostly based on optical cameras because of their low-cost and high-resolution imaging capabilities. Another approach is the use of a radar system, which can obtain certain target characteristics that conventional systems cannot. This technology has attracted considerable attention, especially after the standardization of ultra wide-band (UWB) signals in the United States, because it enables high-resolution ranging of the order of a few centimeters.

A number of UWB radar imaging methods and systems have been proposed that are better than conventional camera-based systems in terms of accuracy and resolution [1-3]. However, conventional radar imaging methods are not sufficiently cost-effective for many applications because most such systems employ a large-scale antenna array, making the entire system considerably more expensive than conventional camera-based systems and hence unsuitable for practical use.

Matsuki et al. proposed a UWB radar imaging method employing only three antennas to estimate the translatory motion of a target [4]. This method assumes that the shape of a local target a circular and optimizes this three kinds of ranging data corresponding to line three degrees of freedom of a circle (the two coordinates of the center position and the radius). However, this method cannot estimate the rotation of the target and thus is difficult to apply to actual data.

This paper presents a UWB radar system that uses five antennas for the simultaneous estimation of the target's shape, translation, and rotation. The performance of the proposed method is evaluated experimentally.

## 2. SYSTEM MODEL

A two-dimensional model is assumed for simplicity, and our objective is to estimate a two-dimensional target shape under this assumption. A five-element linear antenna array is installed, and each antenna element is placed along a straight line at fixed intervals of $\Delta x=0.2 \mathrm{~m}$, as shown in Fig. 1 . The straight line could correspond to a wall or the ceiling of a hallway; the problem could be the imaging of the cross section of a human walking along the hallway.

Each of the antennas is connected to a pulse generator and a receiver via an RF switch. In addition, each antenna is operated as a mono static radar system with modulation to avoid interference among antennas. Any modulation can be employed as long as the base functions are orthogonal to each other so that they realize a multiple access system. Pulses are transmitted from each antenna at time intervals of $\Delta t$, and the resultant echoes are received at the same antenna.

The target is assumed to have an unknown boundary $\left(X_{0}(\xi), Y_{0}(\xi)\right)$, where $0 \leq \xi \leq 2 \pi$ is a parameter. This expression of the target shape $\left(X_{0}(\xi), Y_{0}(\xi)\right)$ needs to satisfy the following


Figure 1: Assumed system model.


Figure 2: Experimental setup for UWB radar imaging.
conditions:

$$
\begin{equation*}
\int_{0}^{2 \pi} X_{0}(\xi) d \xi=0, \quad \int_{0}^{2 \pi} Y_{0}(\xi) d \xi=0 \tag{1}
\end{equation*}
$$

These conditions imply that the centroid of the boundary is at the origin of the assumed coordinates. Under these conditions, we can define rotation independently of the shape of the target. The target undergoes translation $\left(X_{T}(t), Y_{T}(t)\right)$ and rotation $\phi(t)$ with time $t$. The target boundary $(X(\xi, t), Y(\xi, t))$ at time $t$ is expressed as

$$
\left[\begin{array}{l}
X(\xi, t)  \tag{2}\\
(Y(\xi, t)
\end{array}\right]=R(\phi(t))\left[\begin{array}{l}
X_{0}(\xi) \\
Y_{0}(\xi)
\end{array}\right]+\left[\begin{array}{l}
X_{T}(t) \\
Y_{T}(t)
\end{array}\right],
$$

where $R(\phi)$ denotes the rotation matrix

$$
R(\theta)=\left(\begin{array}{cc}
\cos \theta & -\sin \theta  \tag{3}\\
\sin \theta & \cos \theta
\end{array}\right)
$$

The distance between each antenna and the scattering center of the target is measured as $R_{i}(t)(i=1,2, \ldots, 5)$ using the $i$-th antenna at each time step $t=n \Delta t$. The purpose of this paper is to develop a method to estimate the translation $\left(X_{T}(t), Y_{T}(t)\right)$, rotation $\phi(t)$, and the target shape $\left(X_{0}(\xi), Y_{0}(\xi)\right)$ using the ranging data $R_{i}(t)(i=1,2, \ldots, 5)$.

## 3. PROPOSED METHOD

A previous work [4] proposed a method based on fitting using a circular model for estimating the target translation and shape. Because a circle has three degrees of freedom, three antennas were used in their study. However, this method cannot estimate target rotation because a circle is symmetric. We propose a new method that uses an elliptical model instead of a circular one to estimate both the rotation and the translation and thus obtain a target image.

The proposed method estimates the local target shape for each time step $t=t_{n}$ using an elliptical model with five parameters $a, b, x_{0}, y_{0}$, and $\theta$. The ellipse is expressed as

$$
\begin{equation*}
\left(\frac{\cos ^{2} \theta}{a^{2}}+\frac{\sin ^{2} \theta}{b^{2}}\right)\left(x-x_{0}\right)^{2}+\left(\frac{\sin ^{2} \theta}{a^{2}}+\frac{\cos ^{2} \theta}{b^{2}}\right)\left(y-y_{0}\right)^{2}+\sin 2 \theta\left(\frac{1}{a^{2}}-\frac{1}{b^{2}}\right)\left(x-x_{0}\right)\left(y-y_{0}\right)=1, \tag{4}
\end{equation*}
$$

where $a$ and $b$ are the long and short axes of the ellipse, respectively; $\left(x_{0}, y_{0}\right)$ is the center of the ellipse; and $\theta$ is the rotation angle. The distance between the $i$-th antenna and the corresponding scattering center $c_{i}\left(a, b, x_{0}, y_{0}, \theta\right)$ is defined as $r_{i}\left(a, b, x_{0}, y_{0}, \theta\right)$. The variables $c_{i}$ and $r_{i}$ are calculated using the $i$-th antenna position $x_{i}$. The scattering center $c_{i}\left(a, b, x_{0}, y_{0}, \theta\right)$ is equivalent to the point on the ellipse closest to the antenna $x_{i}$ because there is no point closer to the antenna than the foot of perpendicular on a convex curve.

We define a cost function

$$
\begin{equation*}
F_{n}\left(a, b, x_{0}, y_{0}, \theta\right)=\sum_{i 1}^{N_{a}}\left|r_{i}\left(a, b, x_{0}, y_{0}, \theta\right)-R_{i}\left(t_{n}\right)\right|^{2}, \tag{5}
\end{equation*}
$$

where $N_{a}$ is the number of antennas. By minimizing this cost function, we determine the most likely parameter set of an ellipse.

Note that the parameters $a, b, x_{0}, y_{0}$, and $\theta$ should be treated differently because they denote different serial parameters: whereas $a$ and $b$ indicate the target shape, the other parameters indicate the target motion. Using these characteristics, we introduce a smoothing process for $a$ and $b$; we independently estimate the other parameters $x_{0}, y_{0}$, and $\theta$, which are updated at each time step, without smoothing.

The proposed optimization procedure for $a$ and $b$ is expressed as

$$
\begin{equation*}
(\hat{a}, \hat{b})=\arg \min _{(a, b)} \sum_{n 0}^{N_{\text {obs }}} \min _{\left(x_{0}, y_{0}, \theta\right)} F_{n}\left(a, b, x_{0}, y_{0}, \theta\right), \tag{6}
\end{equation*}
$$

where $N_{\text {obs }}$ is the total number of time steps for which the observed data are analyzed using the proposed method. The parameters $(a, b)$ are estimated using a linear search algorithm. For each $(a, b)$, the remaining parameters $\left(x_{0}, y_{0}, \theta\right)$ are optimized using the Levenberg-Marquardt algorithm.

To conduct the optimization, it is necessary to calculate the scattering center points $c_{n, i}$, $i=1,2, \ldots, 5$ for each time step $t=t_{n}$. To find the scattering centers, the optimized elliptical parameters $\left(x_{0 n}, y_{0 n}, a_{n}, b_{n}, \theta_{n}\right)$ and each antenna position $x_{i}$ are used. The scattering center point $c_{n, i}$ is estimated as the foot of the perpendicular line drawn through the $i$-th antenna position. This process can be computed analytically.

The model assumes the fixed parameters $a$ and $b$ throughout the entire data set; however, this does not necessarily mean that the target is modeled as an ellipse because the estimated motion is compensated for in order to obtain the final image, as explained later. In this process, each set of only five points of the final image are simultaneously on the same ellipse.

In this way, the optimization is stabilized because the number of degrees of freedom is reduced from five to three i.e., $\left(x_{0}, y_{0}, \theta\right)$. Although a linear search is relatively time consuming, the procedure does not take long because there are only two parameters to be optimized in this case.

## 4. PERFORMANCE EVALUATION OF THE PROPOSED METHOD USING EXPERIMENTAL DATA

We apply the proposed method to experimental data recorded with the UWB experimental system shown in Fig. 2. In this system, an elliptical cylinder in an anechoic chamber is secured with thin bars on both sides. This cylinder is connected to a rotation actuator that can be electronically operated. In this system, the antennas are scanned instead of the target itself because the received signals are exactly the same as those in the system model assumed in this paper. Note that this system is set up to simulate two-dimensional imaging data. Therefore, the antennas at the top of the figure are scanned in the direction perpendicular to the target cylinder.

The transmitting and receiving antennas are placed close to each other and scanned simultaneously. These UWB antennas have a bandwidth of 2.0 GHz and a center frequency of 3.0 GHz . The bandwidth corresponds to a resolution of 7.5 cm . The antennas are ceramic patch antennas with a beam width wide enough to cover the target; this setup does not significantly differ from the system model assumed for the computer simulation. A wide-band impulse with a width of 80 ps is generated by a signal generator and input to one of the antennas, whereas the signal received by the other antenna is amplified and A/D converted using a sampling oscilloscope. The digitized data are analyzed using the proposed imaging method.

The target has an elliptical section with a long axis of $a=0.15 \mathrm{~m}$ and a short axis of $b=0.10 \mathrm{~m}$. The assumed motion is $X_{T}=X_{0}+v_{x} t, Y_{T}=Y_{T 0}$, and the assumed rotation is $\phi(t)=\phi_{0}+\omega_{\phi}$, where $X_{0}=-0.1 \mathrm{~m}, v_{x}=1.0 \mathrm{~m} / \mathrm{s}, Y_{T 0}=0.435 \mathrm{~m}, \phi_{0}=-1.24 \mathrm{rad}$, and $\omega_{\phi}=3.5 \mathrm{rad} / \mathrm{s}$. The five antennas are spaced at intervals of 10 cm .

Figure 3 shows the signals received by the third antenna (central antenna) under the conditions assumed above. Here, the signals are received every 5 ms and the waveforms appear smooth because they are output via a matched filter. We first extract the peak points of each waveform to calculate $R_{k}(t)$ and then apply the two-step optimization method with the fixed shape parameters. The estimated values of $a$ and $b$ are 15.6 cm and 9.9 cm , respectively. These values are then used to estimate other parameters.

Figure 4 shows the rotation angle estimated using the proposed optimization method with the fixed shape parameters. The figure shows that the proposed method works well even for the experimental data although it has an error of $5.6^{\circ}$ at $t=0.16 \mathrm{~s}$ and $13.8^{\circ}$ at the final time step,


Figure 3: Signal measured with the center antenna of the experimental system.


Figure 4: Actual and estimated rotation angles for experimental data.


Figure 5: Target shape estimated from experimental data using the proposed method.
$t=0.24 \mathrm{~s}$. The solid lines in Fig. 5 show the actual target shapes moving to the right while rotating counter-clockwise. Here, triangles on the $x$-axis show the five antenna positions. The estimated target shape is shown in Fig. 5 with black dots along with the estimated translation $\left(x_{0}, y_{0}\right)$; it is observed that the proposed method accurately estimates the target shape from the experimental data.

## 5. CONCLUSION

In this paper, we proposed a new imaging method for UWB radar using five antennas. The method uses the motion of a target, which comprises both translation and rotation, to obtain a target image by compensating for the estimated motion. The two-step optimization method with fixed parameters was applied to experimental data recorded by a UWB radar experimental system to demonstrate the practicality of the proposed method. The results demonstrate that the proposed method is able to estimate a target shape even if the translation, rotation, and shape of a target are unknown.

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