A Fast Algorithm of 3-Dimensional Imaging for Pulse Radar Systems

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INTRODUCTION 3-dimensional environment measurement is important issue for various applications including rescue robots. The imaging should be reliable if the algorithm affects a person's life. Pulse radar systems have an advantage that they work even in critical situations where optical measurement is not available. Estimating target shapes using data received by a scanned omnidirectional antenna is known as one of ill-posed inverse problems. Many kinds of imaging algorithms have been proposed for this problem. Migration algorithms are well-known as a 3-dimensional imaging algorithm in the field of a seismic prospecting [1]. They work well for most cases but the calculation time cannot be acceptable because they iteratively calculate a great deal of wave equations. Model fitting method is one of effective approaches for this problem [2], [3]. In the model fitting method, target shapes are expressed with parameters, and the parameters are updated to minimize the difference between the observed data and the estimated data. Model fitting method works well to some extent, but they also have a problem concerning calculation time and stability[4], [5].

We proposed a high-speed imaging algorithm for 2-dimensional systems[6]. The algorithm is based on Boundary scattering transform (BST), which is a reversible transform and can be used for direct estimation of target shapes. This transform can be easily extended from 2-dimension to 3-dimension. The calculation time can be considerably reduced compared to conventional algorithms. In this paper, we propose a high-speed 3-dimensional imaging algorithm based on BST.

SYSTEM MODEL We assume a monostatic radar system in this paper. We assume that each target has a uniform complex permittivity, and surrounded by a smooth boundary. An omni-directional antenna is scanned on a plane as in Fig. 1. Pulses are transmitted at a fixed interval and received by the same antenna. The received data is A/D converted and stored in a memory. We estimate target shapes using the data. The transmitted pulse is a mono-cycle pulse, which is suitable for radar systems because it has no DC power. We deal with 3-dimensional problems, and linear polarization. We define r-space as the real space, where targets and the antenna are located. We express r-space with the parameter (x, y, z). All of x, y and z are normalized by λ , which is the center wavelength of the transmitted pulse in vacuum. We assume z > 0 for simplicity. The antenna is scanned on the plane spanned with x-axis and y-axis in r-space. We define s'(X, Y, Z) as the received electric field at the antenna location (x, y, z) = (X, Y, 0), where we define Z with time t and speed of the light c as $Z = ct/(2\lambda)$. We apply a matched filter of transmitted waveform to s'(X, Y, Z). We define s(X, Y, Z) as the output of the filter. We define d-space as the space expressed by (X, Y, Z). We normalize X and Y by λ and Z by the center period of transmitted waveform, respectively. It should be noted that the received data is expressed with (X, Y, Z) in d-space and target shapes are expressed with (x, y, z) in r-space. Transform from (X, Y, Z) to (x, y, z) corresponds to the imaging we deal with in this paper.

BOUNDARY SCATTERING TRANSFORM We define q as the boundary surface which is expressed as a differentiable single-valued function. This assumption includes the case where the target complex permittivity is divided into multiple areas. This assumption is valid for most of artificial targets in the environment for household or rescue robots. We define several sets in order to explain Boundary scattering transform. We define P, which is a subset of d-space, as $P = \{(X, Y, Z) | \partial s(X, Y, Z) / \partial Z = 0, | s(X, Y, Z) | \geq T_s \}$, where T_s is a threshold to prevent picking up noise values. Next, we connect the points close to each other in P. We express each surface as p, which we call a quasi wavefront. We define G as the set of all $p \in P$. Here, we assume that the medium of direct path is vacuum, but the following argument is valid for any uniform media only if the propagation speed of the wave is known. We assume p corresponds to the direct scattered wave of q. By utilizing the relationship between the antenna location and the length of perpendicular line to q from the antenna location, the point (X, Y, Z) on p is expressed as

$$\begin{cases} X = x + z\partial z/\partial x \\ Y = y + z\partial z/\partial y \\ Z = z\sqrt{1 + (\partial z/\partial x)^2 + (\partial z/\partial y)^2}, \end{cases}$$
(1)

where (x, y, z) is a point on q, and we assume z > 0 and Z > 0. We define the transform in Eq. (1) as Boundary Scattering Transform (BST). Fig. 2 shows an example of a target surface and its BST. The figure assumes 2-dimensional system for simplicity. The upper figure shows the target boundary surface in r-domain, and the lower figure is the corresponding quasi wavefront in d-domain, which is the BST of the upper figure.

The inverse transform of BST is given by

$$\begin{cases} x = X - Z\partial Z/\partial X \\ y = Y - Z\partial Z/\partial Y \\ z = Z\sqrt{1 - (\partial Z/\partial X)^2 - (\partial Z/\partial Y)^2}, \end{cases}$$
(2)

which is obtained in the similar way as the 2-dimensional case[6]. The existence of the inverse transform is very meaningful because it can be used for a direct and unique estimation of target boundary shapes. The estimated target boundaries are expressed not as an image but surfaces. This is the advantage and the characteristic of our algorithm. The condition of existence of IBST is differentiability of the quasi wavefront and $(\partial Z/\partial X)^2 + (\partial Z/\partial Y)^2 \leq 1$. This inequality is required because if it is not satisfied, the estimated z using IBST becomes an imaginary number, which is not rational.

PROPOSED ALGORITHM In this section, we propose a 3-dimensional imaging algorithm based on BST and IBST. We have already defined the set P. The procedure of extraction of P is easy because all we should do is to check the derivative of given data. Next, we go on to the procedure of extracting p from P. In an actual procedure, we sequentially connect the points in P which satisfy a required condition. The *i*-th set p_i is determined as follows. The first element of p_i is an arbitrary element of P which is not included in p_1, p_2, \dots, p_{i-1} . The domain I_i for p_i is set to (X, Y) of the first element. The second element of p_i is chosen from P which satisfies $(\partial Z/\partial X)^2 + (\partial Z/\partial Y)^2 \leq 1$. Here, Z should have only one value for the same (X, Y). Then, domain I_i is updated according to the newly chosen element. In this way, we expand the set p_i until there is no other element which can be included into p_i . Next, we calculate $\partial Z/\partial X$ and $\partial Z/\partial Y$ for

an extracted quasi wavefront using a 2-dimensional B-spline smoothing algorithm. Finally, we apply IBST in Eq. (2) to the data and obtain an estimated target surface. Fig. 3 illustrates the outline of the algorithm we propose in this paper.

APPLICATION EXAMPLE We show an application example of the proposed algorithm. The antenna transmits pulses at 51×51 positions with intervals of $\lambda/4$. The assumed target is shown in Fig.4. The inner part of the surface is filled with perfect electric conductor. The scanning plane is z = 3, which means the plane is 1λ apart from the nearest target surface. We obtain the received data s(X, Y, Z) using numerical simulations. First, we extract a quasi wavefront p from s(X, Y, Z) as in Fig. 5. Next, we calculate $\partial Z/\partial X$ and $\partial Z/\partial Y$ in order to apply IBST to the data. The estimated target surface is shown in Fig. 6. As for the calculation time, the proposed algorithm with 51×51 positions takes 0.1 sec with a single Xeon 2.8GHz processor for the entire reconstruction.

CONCLUSIONS In this paper, we have proposed a new 3-dimensional imaging algorithm based on BST for pulse radar systems. BST is known as a reversible transform between target surfaces and received wave delay for 2-dimensional systems. We have extended BST to 3-dimensional systems and apply it for 3-dimensional imaging. First, we have shown the extended transform and its inverse transform. Secondly, we have proposed a 3-dimensional imaging algorithm based on BST. Finally, we have shown an application example of the proposed algorithm. We have clarified that the proposed algorithm's calculation time is considerably short compared to conventional algorithms, so that the proposed algorithm can be readily implemented to realtime applications. Additionally, the estimated target shape is accurate enough on the condition that the directly scattered waveform can be obtained. In this paper, we have shown an application example without noise. Investigating the performance of the proposed algorithm under noisy conditions will be an important future task.

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Fig. 1. System model and antenna scanning.

Fig. 4. True target shape.



Fig. 2. An example of Boundary Scattering Transform.



Fig. 3. The outline of the proposed algorithm.



Fig. 5. Extracted quasi wavefront.



Fig. 6. Estimated target shape. Computation time for the reconstruction is $0.1~{\rm sec.}$