High Resolution Ultrasound Imaging Using Frequency Domain Interferometry

--- Suppression of Interference Using Adaptive Frequency Averaging ---

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We have reported that frequency domain interferometry (FDI) imaging with the Capon method has the potential to acquire high range-resolution ultrasound images. The reported method employed uniform frequency averaging to suppress coherent interferences. In atmospheric radar imaging, adaptive averaging has proposed to perfectly suppress coherent interference. In the present study, we applied adaptive frequency averaging to the FDI imaging method and investigated its image quality in a simulation study, where the –6 dB bandwidth of the echo returned from an interface was 2.6 MHz. When two targets 0.1 mm apart from each other were located in a region of interest the FDI imaging method using uniform frequency averaging succeeded in estimating the interface ranges correctly; however, the estimated echo intensity was 6.3 dB lower than the true echo intensity. In contrast, the FDI imaging method using adaptive frequency averaging successfully estimated the interface ranges, and the estimated echo intensity was only 0.049 dB higher than the true one. These results indicate that the FDI imaging method using adaptive frequency averaging has excellent accuracy in the measurement of echo intensity, under the condition that the echo waveform returned from each target is the same as the waveform of the reference signal.

Keywords: ultrasound imaging, high-resolution, vascular ultrasound, frequency domain interferometry, frequency averaging

1. Introduction

Cardiovascular disease (CVD) is one of the main causes of mortality worldwide. Vascular ultrasound is a non-invasive imaging method that acquires images of arterial walls. The measurement of carotid artery thickness using ultrasonography (US) is considered to be useful in detecting early atherosclerosis (1)-(3). Therefore, improvement in the range resolution of vascular ultrasound is expected to enable the detection of CVD at an earlier stage.

We have reported that the frequency domain interferometry (FDI) imaging method with the Capon method has the potential to acquire high range-resolution vascular ultrasound images (4)-(6). In US, the echoes returned from different targets are strongly correlated (7)(8). Since the FDI method with a power minimizing technique, such as the Capon method does not work when the desired signal and interferences are correlated with each other (9)(10), we employed uniform frequency averaging to suppress the correlation between the desired signal and interference. However, uniform frequency averaging cannot suppress the correlation perfectly, and this suppression depends on the bandwidth used for the uniform frequency averaging. When the suppression of the correlation is insufficient, the accuracy of the FDI imaging method in echo intensity estimation deteriorates (11).

In atmospheric radar imaging, adaptive averaging has been proposed to more effectively suppress the correlation between the desired signal and interference (12)(13). Adaptive averaging selects the optimum weights adaptively to perfectly suppress the correlation. In the present study, we applied adaptive frequency averaging to the FDI method with the Capon method, and evaluated the effect of adaptive frequency averaging on ultrasound imaging.

2. Materials and Methods

We applied the proposed method to each received signal in a scan line separately. The proposed method is based on the FDI imaging method with the Capon method using the whitening technique. We briefly describe the FDI imaging method with the Capon method, and subsequently explain the frequency averaging techniques.

2.1 FDI Imaging Method with the Capon Method

The FDI imaging method with the Capon method assumes that the frequency components of a signal returned from a target have the same intensity. Since US utilizes broadband signals, we should employ the whitening technique to correct the intensity of all frequency components uniformly (14). The received RF signal in the frequency domain after whitening is expressed as follows:

\[
X_{HF} = [X_{H1} \quad X_{H2} \quad \ldots \quad X_{HN}]^T, \quad \text{.................................. (1)}
\]

\[
X_{H} = X_{p} X_{r}^* \sqrt{\frac{1}{|Y_{k}|^2 + \eta}}, \quad \text{.................................. (2)}
\]

where \(\eta\) is a constant term for stabilization, \(N\) is the number of frequency component samples used for imaging, \([\cdot]^T\) denotes the
transpose, and $X_{1h}, X_{2h},$ and $X_{0h}$ are the $i$-th frequency component of a received RF signal after whitening, the $i$-th frequency component of a received RF signal before whitening, and the $j$-th frequency component of a reference RF signal without whitening, respectively.

The estimated intensity of the FDI imaging method with the Capon method at the depth $r/2$ is given by:

$$P_{cap}(r) = \frac{1}{C^T R^{-1} C}. \quad \text{(3)}$$

$$R = X_{1h} X_{2h}^T. \quad \text{(4)}$$

$$C = [e^{ikr} \quad e^{ikr} \cdots e^{ikr}]^T. \quad \text{(5)}$$

where $k_i$ is the $i$-th wavenumber of the frequency components of the received RF signal, and $[ ]^T$ denotes the conjugate transpose.

2.2 Uniform Frequency Averaging

When a received signal consists of multiple echoes, the $(i_1, i_2)$ element of a covariance matrix $R$ is given by:

$$R_{i_1, i_2} = X_{1i_1} X_{2i_2}^T$$

$$= \sum_l \sum_j \alpha_l \exp(-j k_i r_j) \sum_l \sum_j \alpha_j \exp(j k_i r_j)$$

$$= \sum_l \sum_j \alpha_l \exp\left(-j (k_i - k_j) r_j \right) + \sum_l \sum_j \alpha_j \exp(g_x + g_y),$$

$$g_x = \exp\left(-j (k_i - k_j) r_j \right), \quad \text{(6)}$$

$$g_y = \exp\left(-j (k_i - k_j) r_j + j k_i \Delta r \right), \quad \text{(7)}$$

$$\Delta r = r_j - r_i, \quad \text{(8)}$$

where $l_1$ and $l_2$ ($l_1 < l_2$) are target numbers, $\alpha_l$ is the amplitude of the echo returned from the $l$-th target and $r_j/2$ is the target depth of the $l$-th target. We introduce the assumption that the amplitude of each frequency component of an echo after whitening is independent of its frequency. The second term of the last line of Eq. (6) is the cross-correlation term between different targets.

Since the FDI imaging method with the Capon method assumes that there is no correlation between the desired signal and the interference, the correlation term should be sufficiently suppressed. In atmospheric radar, the cross-correlation term is suppressed by the temporal averaging. However, in vascular ultrasound, targets are not stationary, resulting in difficulty in employing temporal averaging.

The first term of the last line of Eq. (6), the auto-correlation term, is constant when $i_1 - i_2$ is constant. Since the averaging of a covariance matrix along the diagonal direction suppresses the cross-correlation term, frequency averaging averages sub-matrices along the diagonal direction, as shown in Fig. 1.

$$R_{i_1, i_2} = \sum_v v_n R_{n, n}$$

subject to $\sum_n v_n = 1, \quad \text{(11)}$

$$R_{v_{i_1, i_2}} = R_{v_{i_1-1, i_2-1}}. \quad \text{(12)}$$

where $R_{v}$ is a covariance matrix after frequency averaging, $v_n$ is a real number, $M$ is the number of the sub-matrices used for frequency averaging, $R_{v_{i_1, i_2}}$ and $R_{v_{i_1-1, i_2-1}}$ are the $(i_1, i_2)$ elements of a covariance matrix $R$ and a $m$-th sub-matrix $R_{v_{m}}$, respectively.

Uniform frequency averaging employs a constant weight $v_n = 1/M^0$. The covariance matrix after uniform frequency averaging $R_{UA}$ is given by:

$$R_{UA_{i_1, i_2}} = \sum_j \sum_k \alpha_l \exp\left(-j (k_i - k_j) r_j \right) + \sum_l \sum_k \alpha_j \exp\left(g_x + g_y, \right),$$

$$\Delta r = r_j - r_i, \quad \text{(9)}$$

where $R_{UA_{i_1, i_2}}$ is the $(i_1, i_2)$ element of the covariance matrix after frequency averaging $R_{UA}$. $F_s$ is the bandwidth used for frequency averaging, $F_c$ is the $i$-th frequency and $c$ is the sound velocity. These equations indicate that the cross-correlation term is suppressed in proportion to a sinc function of $\pi F_s \Delta r/c$. In the case of $F_s \Delta r/c < 0.5$ the FDI imaging method using uniform frequency averaging does not suppress the cross-correlation term sufficiently, causing deterioration in its performance. When the bandwidth for frequency averaging $F_s$ is 2 MHz, the FDI imaging method using uniform frequency averaging drops into the severe situation under the condition of the target interval $\Delta r/2 < 0.2 \text{ mm}$.

The estimated intensity of the FDI imaging method with the Capon method using uniform frequency averaging is given by:

$$P_{capUA}(r) = \frac{1}{C^T R^{-1} C + \eta^T E \cdot C^T}, \quad \text{(16)}$$

$$C = [e^{ikr} \quad e^{ikr} \cdots e^{ikr}]^T, \quad \text{(17)}$$

where $\eta^T E$ is a diagonal loading matrix used to obtain the inverse matrix $R_{UA}^{-1}$ stably.

2.3 Adaptive Frequency Averaging

Adaptive frequency averaging employs the optimum weight $v_n$ to effectively suppress the cross-correlation term. When the cross-correlation term is perfectly suppressed, the $(i_1, i_2)$ element of the covariance matrix after frequency averaging is given by:

$$R_{h_{i_1, i_2}} = \sum_j \sum_k \alpha_l \exp\left(-j (k_i - k_j) r_j \right), \quad \text{(18)}$$
Therefore, the covariance matrix with no cross-correlation term becomes a Toeplitz matrix. Adaptive frequency averaging selects the weight \( v_m \) adaptively so as to approximate the covariance matrix to a Toeplitz matrix\(^{(10)}\).

The \((i_1, i_2)\) element of the covariance matrix after frequency averaging is derived from Eqs. (10), (11) and (12).

\[
R_{\text{Av},i_1i_2} = \sum_{m=1}^{M} v_m R_{(i_1-1)(i_2-1)m}. \tag{19}
\]

The measure of deviation of the covariance matrix from a Toeplitz form is given by:

\[
\varepsilon = \frac{1}{L-1} \sum_{i=1}^{L-1} \left| R_{\text{Av},i} - \mathbf{R}_{\text{Av}} \right|^2. \tag{20}
\]

where \( L = N - M + 1 \) is the size of the covariance matrix after frequency averaging, and \( \mathbf{R}_{\text{Av}} \) is the average along the \( i\)’th sub-diagonal of the \( m\)’th sub-matrix. Eq. (20) can be rewritten as follows:

\[
\varepsilon = \mathbf{V}^{\dagger} \mathbf{R}_{\text{Av}} \mathbf{V}, \tag{22}
\]

where \( \mathbf{V} = \begin{bmatrix} v_1 & v_2 & \cdots & v_M \end{bmatrix}^\dagger, \tag{23} \)

\[
\mathbf{R}_{\text{Av}} = \sum_{i=1}^{L-1} \mathbf{R}_i \tag{24}
\]

The minimization of \( \varepsilon \) under the constraint of Eq. (11) is expressed by

\[
\min_{\mathbf{V}} \mathbf{V}^{\dagger} \mathbf{R}_{\text{Av}} \mathbf{V}, \tag{27}
\]

subject to \( \mathbf{V}^{\dagger} \mathbf{I} = 1, \tag{28} \)

where

\[
\mathbf{I} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^\dagger. \tag{29}
\]

This problem can be solved by the application of Lagrange multiplier methods. The optimum vector \( \mathbf{V}_{\text{opt}} \) for the solution to (27) is given by:

\[
\mathbf{V}_{\text{opt}} = \left( \mathbf{R}_{\text{Av}} + \eta \mathbf{E} \right)^{-1} \mathbf{I}, \tag{30}
\]

subject to \( \mathbf{V}^{\dagger} \mathbf{I} = 1, \tag{31} \)

where \( \eta \mathbf{E} \) is a diagonal loading matrix to obtain the inverse matrix \( \mathbf{R}_{\text{Av}}^{-1} \) stably. The employment of a large \( \eta \mathbf{E} \) approximates \( \mathbf{V}_{\text{opt}} \) to a constant weight set employed in uniform frequency averaging.

The estimated intensity of the FDI imaging method with the Capon method using adaptive frequency averaging is given by:

\[
\text{Intensity} = \frac{1}{C_{\text{A}} \left( \mathbf{R}_{\text{Av}} + \eta \mathbf{E} \right)^{-1} \mathbf{C}_{\text{A}}}, \tag{32}
\]

where \( \mathbf{R}_{\text{Av}} = \sum_{m=1}^{M} v_{\text{opt}} \mathbf{v}_m \mathbf{v}_m^{\dagger}. \tag{33} \]

\[\text{where } v_{\text{opt}} \text{ is the } m\text{-th element of the optimum vector } \mathbf{V}_{\text{opt}}.\]

**2.4 Simulation Setup** In the simulation study, we employed the echo returned from an interface between 20% gelatin and 4% agar acquired by a Hitachi EUB-8500 (Hitachi, Tokyo, Japan) US device with a 7.5 MHz linear array. The sampling frequency of the acquired RF data is 30 MHz. Fig. 2 shows the envelope and waveform of an echo returned from a horizontal interface. Since the \(-6\) dB bandwidth of an echo returned from a horizontal interface is 2.6 MHz, the range resolution of a conventional imaging method using the intensity of the received signal is about 0.29 mm. In our study, we supposed the sound velocity \( c \) to be 1500 m/s.

We assume that a received signal is given by:

\[
S(g, t) = \sum_{l} S(l, g, t - t_l / c), \tag{34}
\]

where \( S(g, t) \) is an echo returned from an interface whose slope angle is \( \gamma \) degree, and \( r_l / 2 \) is the depth of the \( l\)’th target. In the present study, we employed 6 echoes returned from interfaces with slope angles of 0, 2, 4, 6, 8 and 10 degrees. We used the echo from a horizontal interface as the reference signal.

The optimum stabilization term \( \eta \) at whitening is equal to the noise intensity of each frequency component. In our study, we employed the average intensity of the frequency components from 11 to 14 MHz as the value of \( \eta \).

Since the loading of \( \eta \mathbf{E} \) is equivalent to the addition of white noise to the received signal, the employment of a small \( \eta \) is desirable. We employed \(-40\) dB of the average value of the diagonal terms of \( \mathbf{R}_{\text{Av}} \) as the value of \( \eta \).

We used 54 frequency components from 5 to 9 MHz, where the frequency interval was 75 kHz. The bandwidth for frequency averaging was 2 MHz. We measured the depth from 10 to 20 mm, where the range interval was 0.01 mm.

**3. Results**

**3.1 Targets of Horizontal Interfaces** First, we investigated the performance of the FDI imaging method under
the condition that a couple of horizontal interfaces are located in a region of interest (ROI). In this case, the echo waveform returned from each interface is the same as the waveform of the reference signal. When the target interval is less than 0.3 mm, the conventional method using the intensity of the received signal could hardly depict the two targets separately, as shown in Fig. 3.

The FDI imaging method with the Capon method successfully estimated all the target depths correctly, as shown in Figs. 4 and 5. The FDI imaging method using uniform frequency averaging estimated the echo intensity returned from interfaces at 6.3, 1.6 and 0.19 dB lower than the true echo intensity when the target intervals were 0.1, 0.2 and 0.3 mm, respectively. The performance of the FDI imaging method using uniform frequency averaging in echo intensity estimation deteriorates severely when the target interval is 0.2 mm and less. Since we used a bandwidth of 2 MHz for uniform frequency averaging, this result is consistent with the consideration detailed at the end of section 2.2.

The FDI imaging method using adaptive frequency averaging estimated the echo intensity returned from interfaces at 0.049, 0.10 and 0.030 dB higher than the true echo intensity when the target intervals were 0.1, 0.2 and 0.3 mm, respectively. Here, we employed –40 dB of the average value of the diagonal terms of $\mathbf{R}_e$ as the value of $\eta_V$. This result suggests that adaptive frequency averaging is effective for the accurate estimation of echo intensity.

Fig. 6 shows the estimated intensity of the FDI imaging method using uniform frequency averaging and adaptive frequency averaging, where six horizontal interfaces are located in the ROI and the target interval is 0.3 mm. The performance of the FDI imaging method using adaptive frequency averaging in echo intensity estimation is superior to that using uniform frequency averaging. This result indicates that adaptive frequency averaging has an excellent accuracy in measuring echo intensity under the condition that the echo waveform returned from each target is the same as the waveform of the reference signal.

3.2 Targets of Sloped Interfaces We investigated the performance of the FDI imaging method under the condition that sloped interfaces are located in a ROI. For adaptive frequency
from the waveform of the reference signal, elements of echoes of various waveforms.

This result indicates that adaptive frequency averaging is almost equivalent to that using uniform frequency averaging. Employing 0 dB diagonal loading for adaptive frequency averaging of large slope angles. The performance of the FDI imaging method averaging in estimating the echo intensity returned from interfaces imaging method is inferior to that using uniform frequency averaging. Six interfaces are located in the ROI, and their slope angles are 0, 2, 4, 6, 8 and 10 degrees, respectively.

Fig. 7 shows that the estimated intensity of the FDI imaging method using uniform frequency averaging and adaptive frequency averaging. Six interfaces are located in the ROI, and their slope angles are 0, 2, 4, 6, 8 and 10 degrees. The employment of –40 dB diagonal loading for adaptive frequency averaging caused the appearance of false images. The employment of –20 dB diagonal loading for adaptive frequency averaging suppressed the appearance of false images; however, the performance of the imaging method is inferior to that using uniform frequency averaging in estimating the echo intensity returned from interfaces of large slope angles. The performance of the FDI imaging method employing 0 dB diagonal loading for adaptive frequency averaging is almost equivalent to that using uniform frequency averaging. This result indicates that adaptive frequency averaging is unsuitable under the condition where a received signal consists of echoes of various waveforms.

4. Discussion

The weighting vector \( \mathbf{V} \) should be composed of real numbers, as this constraint ensures that the auto-correlation terms, each of which is given by the first term of the last line of Eq. (6), are summed coherently and the information related to the target ranges is preserved. When the echo waveform returned from each target was the same as the waveform of the reference signal, elements of \( \mathbf{e}_j^T \mathbf{e}_j \) were real numbers, as shown in Fig. 8(a). Therefore, adaptive frequency averaging employing the optimum vector given by Eq. (30) successfully made the covariance matrix of a Toeplitz form. This means that adaptive frequency averaging sufficiently suppresses the cross-correlation term between different targets, resulting in accurate echo intensity estimation.

When the echo waveform returned from a target was different from the waveform of the reference signal, elements of \( \mathbf{e}_j^T \mathbf{e}_j \) had imaginary parts, as shown in Fig. 8(b). The appearance of imaginary parts is mainly caused at whitening. Since the echo waveform returned from a sloped interface is different from the waveform of the reference signal, whitening fails in correcting the intensity of all frequency components uniformly. The appearance of imaginary parts at the elements of \( \mathbf{e}_j^T \mathbf{e}_j \) indicates that the covariance matrix does not become a Toeplitz matrix after adaptive frequency averaging. Therefore, the performance of the FDI imaging method using adaptive frequency averaging severely deteriorates under the condition where various sloped interfaces are located in a ROI.

This study demonstrates the effectiveness and limitation of adaptive frequency averaging. The FDI imaging method using adaptive frequency averaging does not work stably when the received signal consists of echoes of various waveforms; however, it delivers excellent performance under the condition that the echo waveform returned from each target is the same as the waveform of the reference signal. Therefore, we should locate a small ROI at an appropriate position to satisfy the constraint that the waveform of each echo included in the ROI is same as that of a reference signal. For example, a proper ROI may allow the imaging method for the estimation of carotid artery thickness.

5. Conclusion

In this study, we investigated the FDI imaging method with the Capon method using adaptive frequency averaging. When a received signal consists of echoes of various waveforms, the FDI imaging method using adaptive frequency averaging does not work stably. However, when two horizontal interfaces 0.1 mm apart are located in the ROI, the FDI imaging method using adaptive frequency averaging successfully estimates the interface ranges and the estimated echo intensity is 0.049 dB higher than the true one. Our study indicates that the FDI imaging method using adaptive frequency averaging has an excellent accuracy in measuring echo intensity as compared with the FDI imaging method using uniform frequency averaging, under the condition that the echo waveform returned from each target is the same as the waveform of the reference signal.

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